Q: Using economic theory explain the rationale for having a fixed monthly tariff, as well as charges for each call, in the market for mobile phones.

Solution :

1. Rational consumer choice theory begins with a budget constraint or opportunity set.
    a. The slope of the constraint shows the relative price ratio of the two goods under consideration. This is the rate at which the two goods can be exchanged in the marketplace.
    b. The location of the budget constraint shows the amount of income that is available.
    c. One good can be used as a composite good that represents income spent on all goods other than X.
2. Consumer preference patterns are the next building block of consumer theory.
    a. Consumer preference orderings must be complete, transitive, and more must be preferred to less.
    b. These qualities lead to indifference curves that are negatively sloped, nonintersecting, and continuous.
    c. The slope of the indifference curves shows the rate at which the consumer would like to exchange one good for the other. This is called the marginal rate of substitution.
    d. A diminishing marginal rate of substitution is common and results in an indifference curve that is convex to the origin.
    e. Perfect substitutes have straight-line indifference curves and perfect complements have L-shaped curves.
3. Maximizing a consumer's utility requires that the marginal rate of substitution equal the price ratio of the goods.
4. Where the marginal rate of substitution cannot equal the price ratio, a corner solution exists and all income is spent on one good.
5. A utility function analysis will result in the same conclusions that the indifference curve process has set forth.

Now we start our work with current problem. Supposing that a consumer/customer is able to purchase a commodity Q_1 (mobile with monthly tariff) or Q_2 (mobile with per call charges) or combination of Q_1 and Q_2, depending upon the income (or amount) which he desires to spend on these two commodities. He also desires to choose such combinations of Q_1 and Q_2 so as to derive the highest level of satisfaction from the amount spent. If the amount which he want to spend is given to be y and the prices of q_1 (whole or part of Q_1) and q_2 (whole or part of Q_2) are p_1 and p_2; the problem will be that of maximization of consumer's utility function:

$$U = f(q_1, q_2)$$

Subject to income constraint: $y = p_1q_1 + p_2q_2$

Applying the Langrange's method to obtain the amount of q_1 and q_2 which would give maximum satisfaction to consumer under income constraint.

$$Z = U(q_1, q_2) + \lambda(y - p_1q_1 - p_2q_2)$$

or $$Z = U(q_1, q_2) - \lambda(p_1q_1 + p_2q_2 - y)$$

First order condition:

$$\frac{\partial Z}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda p_1 = 0$$

$$\frac{\partial Z}{\partial q_2} = \frac{\partial U}{\partial q_2} - \lambda p_2 = 0$$

$$\frac{\partial Z}{\partial \lambda} = y - p_1q_1 - p_2q_2 = 0$$

Letting $\frac{\partial U}{\partial q_1} = f_1$ and, $\frac{\partial U}{\partial q_2} = f_2$, the above equations can be written as

$$\frac{\partial Z}{\partial q_1} = f_1 - \lambda p_1 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

$$\frac{\partial Z}{\partial q_2} = f_2 - \lambda p_2 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$$\frac{\partial Z}{\partial \lambda} = y - p_1q_1 - p_2q_2 = 0 \quad \ldots\ldots\ldots\ldots \text{(3)}$$

Solution of the above three equations would give us a point which the utility would either be maximum or minimum.

$$\frac{f_1}{p_1} = \frac{f_2}{p_2} = \lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

but since $f_1 = MUq_1$ and $f_2 = MUq_2$ so the (4) becomes

MUq_1/ price of Q_1 = MUq_2/ price of Q_2 = λ …….. (5)

The (4) equation may be written as

$$\frac{f_1}{f_2} = \frac{p_1}{p_2} = \lambda \text{ or } -\frac{f_1}{f_2} = -\frac{p_1}{p_2} \quad \text{.................. (6)}$$

[Note : the budget constraint y = $p_1q_1 + p_2q_2$ can be written as $q_2 = \left(-\frac{p_1}{p_2}\right)q_1 + \frac{y}{p_2}$

so the ratio $-\frac{p_1}{p_2}$ represent the slope of the budget line]

Second order condition would enable us to say whether the function $U = f(q_1, q_2)$ would be maximum at that point. $U = f(q_1, q_2)$ has the maximum value at point M ($\frac{f_1}{f_2} = \frac{p_1}{p_2}$) if and only of bordered Hessian : $|\overline{H}| > 0$.

Apply the second oreder condition to equation (1), (2) and (3) using bordered hessian technique

$$|\overline{H}| = \begin{vmatrix} 0 & -p_1 & -p_2 \\ -p_1 & f_{11} & f_{12} \\ -p_2 & f_{12} & f_{22} \end{vmatrix} > 0$$

or

$$|\overline{H}| = (-)(f_{11}.p_2^2 - 2f_{12}.p_1.p_2 + f_{22}.p_1^2) > 0$$

It is a positive quantity if and only if the indifferent curves are convex downward at M.

If the combinations of these plans are not possible than the solutions are called end-points and consumer can calculate the benefit directly at either end of budget line where it cuts axes.