A flat uniform circular disk (radius=2.00 m; mass=100 kg) is initially stationary. The disk is free to rotate in the horizontal plane about a frictionless axis perpendicular to the center of the disk. A 40 kg person, standing 1.25 m from the axis, begins to run on the disk in a circular path and has a tangential speed of 2.00 m/s relative to the ground. Find the resulting angular speed of the disk (in rad/s)

a. 0.500 rad/s
b. 2.00 rad/s
c. 1.25 rad/s
d. 0.250 rad/s
e. 1.00 rad/s

<u>Solution</u> :

Here there is no external torque acting on the system thus we can apply the law of conservation of angular momentum
So momentum is conserved in this case.
Angular momentum of the man = $I\omega$
Where I = Inertia of the man about the axis of rotation
or $I = M r^2 = 40 * 1.25*1.25 = 62.5$
ω = Angular velocity of the man, that can be calculated as follows
Tangential velocity of man = v = 2m/s
Circumference of the circle about which man is running = $2\pi r = 2 * 3.14 * 1.25 = 7.85$m
So time taken to describe this circle is $t = 2\pi r / v$
Now angle described in 1 revolution $\theta = 2\pi$ radians
This angle is subtended in time $t = 2\pi r / v$
Thus angular speed = $\omega = \theta/t = 2\pi (v/ 2\pi r) = v/r = 2/1.25 = 1.6$ rad/s
So angular momentum of man = $I\omega = 62.5 * 1.6 = 100$

To conserve the angular momentum before and after the man starts running we have
Angular momentum of disk = angular momentum of the man
or $I_{disk}\ \omega_{disk} = 100$
$I_{disk} = M_{disk}R^2/2 = 100*2*2 / 2 = 200$

Thus $200\ \omega_{disk} = 100$
or $\omega_{disk} = ½ = 0.5$ rad/s