The following log-linear demand curve for a price-setting firm is estimated using the ordinary least squares method:

$$Q = aP^b M^c P_R^d$$

where *Q* and *P* are the quantity and price respectively for good *X, M* is consumer income, and P_R is the price of good *R.*
The estimation results are presented below:

DEPENDENT VARIABLE:	LNQ	R-SQUARE	F-RATIO	P-VALUE ON F	
OBSERVATIONS:	64	0.8464	110.25	0.0001	
VARIABLE		PARAMETER ESTIMATE	STANDARD ERROR	T-RATIO	P-VALUE
INTERCEPT		5.65	3.20	1.77	0.0825
LNP		-1.02	0.59	-1.73	0.0890
LNM		0.45	0.22	2.05	0.0452
LNPR		-2.0	0.75	-2.67	0.0098

a. Express the estimated demand equation in logarithms.

Ans.

Taking log on both sides of the eq $Q = aP^b M^c P_R^d$ we get

LnQ = Ln a + b lnP + c LnM + dLnPR

Substituting the estimates of coefficients obtained by ordinary least squares method our regression becomes

LnQ = 5.65 -1.02 lnP + 0.45 LnM – 2 LnPR

b. Is *X* a normal or an inferior good? And how are goods *X* and *R* related? Explain.

Ans. X is a normal good because as income increases demand also increases.

As price of goods R increases the Quantity of goods X decreases showing that quantity of X and price of goods R are negatively correlated.

Which of the parameter estimates are statistically significant at the 5 percent level?

Ans. parameter estimates of variables LnM + LnPR are statistically significant at the 5 percent level as their p values are < 0.05 .

c. Estimate the own-price elasticity for good *X,* the cross-price elasticity for goods *X* and *R,* and the income elasticity for good *X.*

Ans.

own-price elasticity refers to changes in quantities due to changes in the price of that good.

Ans. Quantity demanded = $Q = aP^b M^c P_R^d$ (1)

When price were to increase by 1% P= 1.01P

$Q' = a(1.01P)^b M^c P_R^d$ (2)

Dividing (2) by (1) we get

$\frac{Q'}{Q} = 1.01^b = 1.01^{-1.02} = 0.9899$

Thus When price were to increase by 1% , quantity demanded decreases by 1.01%.So own-price elasticity for good *X* = -1.01/1 = -1.01

The cross-price elasticity for goods *X* and *R*

Quantity demanded = $Q = aP^b M^c P_R^d$ (1)

When price of R were to increase by 1% PR'= 1.01PR

$Q' = aP^b M^c (1.01P_R)^d$ (2)

Dividing (2) by (1) we get

$\frac{Q'}{Q} = 1.01^d = 1.01^{-2} = 0.9803$

Thus When price were to increase by 1% , quantity demanded decreases by 1.97%.So own-price elasticity for good *X* = -1.97/1 = -1.97

Income elasticity for good *X*

Quantity demanded = $Q = aP^b M^c P_R^d$ (1)

When INCOME increase by 1% M'= 1.01M

$Q' = aP^b (1.01M)^c P_R{}^d$ (2)

Dividing (2) by (1) we get

$\frac{Q'}{Q} = 1.01^C = 1.01^{0.45} = 1.0045$

Thus When price were to increase by 1% , quantity demanded increases by 0.45%.Thus Income elasticity for good X = *0.45*

d. Holding all other things constant, if household income were to fall by 22%, what would we expect to happen to quantity demanded? Explain.

Ans. Quantity demanded = $Q = aP^b M^c P_R^d$ (1)

When household income were to fall by 22% M'= 0.78M

$Q' = aP^b (0.78M)^c P_R^d$ (2)

Dividing (2) by (1) we get

$\frac{Q'}{Q} = 0.78^c = 0.78^{0.45} = 0.8942$

Thus quantity demanded is reduced by 10.58%.

e. Holding all other things constant, if own price were to increase by 22%, what would we expect to happen to quantity demanded? Explain.

Ans.

Proceeding in the same way as in (e) we get

$\frac{Q'}{Q} = 1.22^b = 1.22^{-1.02} = 0.8164$

Thus quantity demanded is reduced by 18.36%.

f. Holding all other things constant, if the price of *R* were to fall by 8%, what would we expect to happen to quantity demanded? Explain.

Ans.

Proceeding in the same way as in (e) we get

$\frac{Q'}{Q} = 0.92^d = 0.92^{-2} = 1.1815$

Thus quantity demanded is increased by 18.15%.