

A certain airplane has two independent alternators to provide electrical power. The probability that a given alternator will fail on a 1-hour flight is .02. What is the probability that

(a) both will fail?

Ans.

$$P(\text{Alternator fails}) = 0.02$$

$$P(\text{Alternator does not fail}) = 0.98$$

A = 1st alternator fails

B = 2nd alternator fails

$$P(A) = P(B) = 0.02$$

$$P(AB) = P(A) \cdot P(B) \quad \text{as } A \text{ \& } B \text{ are independent.}$$

$$P(\text{both fail}) = P(1^{\text{st}} \text{ alternator fails and } 2^{\text{nd}} \text{ alternator also fails}) = P(AB) = P(A) \cdot P(B) = 0.02 \cdot 0.02 = 0.004$$

(b) Neither will fail?

Ans.

$$P(\text{Alternator fails}) = 0.02$$

$$P(\text{Alternator does not fail}) = 0.98$$

A = 1st alternator fails

$\bar{A}$ = 1st alternator does not fail

B = 2nd alternator fails

$\bar{B}$ = 2nd alternator does not fail

$$P(A) = P(B) = 0.02$$

$$P(\bar{A}) = P(\bar{B}) = 1 - 0.02 = 0.98$$

$$P(\text{Neither will fail}) = P(1^{\text{st}} \text{ alternator does not fail and } 2^{\text{nd}} \text{ alternator does not fail}) = P(\bar{A}\bar{B}) =$$

$$P(\bar{A})P(\bar{B}) = 0.98 \cdot 0.98 = 0.9604$$

(c) One or the other will fail? Show all steps carefully.

Ans.

$$P(\text{Alternator fails}) = 0.02$$

$$P(\text{Alternator does not fail}) = 0.98$$

$$P(A) = P(B) = 0.02$$

$$P(\bar{A}) = P(\bar{B}) = 1 - 0.02 = 0.98$$

A = 1st alternator fails

B = 2nd alternator fails

$$\begin{aligned} P(\text{One or the other will fail}) &= P(1^{\text{st}} \text{ Alternator fails} \cdot P(2^{\text{nd}} \text{ alternator does not fail}) + P(2^{\text{nd}} \\ \text{Alternator fails}) \cdot P(1^{\text{st}} \text{ alternator does not fail}) &= P(A)P(\bar{B}) + P(\bar{A})P(B) = 0.02 \cdot 0.98 + \\ 0.02 \cdot 0.98 &= 0.0392 \end{aligned}$$